Microlensing Probes of Dark Matter Substructure

Fabian Schmidt
Princeton
Motivation

• Standard CDM predicts hierarchical DM substructure down to \( \sim M_{\text{Earth}} \)

• Lower mass cutoff determined by details of DM production - but also tidal disruption...

• How can we detect / constrain this structure in the \(< 10^8 \, M_{\odot}\) range?

• Key parameter: \( DM \) density within substructure
Basic picture
Basic picture

Lensing effect strongly exaggerated...
Lensing observables

• Time (Shapiro) delay \( \propto \int d\chi \Phi \)

• Doppler shift \( \propto \nabla \Phi \)

• Astrometric perturbations \( \propto \int d\chi \frac{\chi_s - \chi}{\chi_s} \nabla \Phi \)

• Magnification \( \propto \int d\chi \frac{\chi}{\chi_s} (\chi_s - \chi) \nabla^2 \Phi \)

Weak lensing approx (cf later)

\( \Phi \) : grav potential of lens in comoving frame

("moving screen" approx.)
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Motion of source/observer

Weak lensing approx (cf later)

\( \Phi \): grav potential of lens in comoving frame

(“moving screen” approx.)
Relevant scalings

- Potential at scale radius:
  \[ \Phi_s = \Phi(r_s) \simeq 3 \cdot 10^{-12} \left( \frac{M_s}{10^6 M_{\odot}/h} \right)^{2/3} \Delta_s^{1/3} \]

- where
  \[ M_s = M(< r_s) \quad \Delta_s = \frac{\rho(< r_s)}{\bar{\rho}} \]

  subhalo mass  subhalo density

Note: no density profile assumed
Relevant scalings

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- where

\[ M_s = M(< r_s) \quad \Delta_s = \frac{\rho(< r_s)}{\bar{\rho}} \]

- Scale radius in terms of density:

\[ r_s = 14h^{-1}\text{kpc} \left( \frac{M_s}{10^6 M_\odot/h} \right)^{1/3} \Delta_s^{-1/3} \]

Note: no density profile assumed
Relevant scalings

• For a moving lens, we have

\[
\frac{r_s}{v_{\text{rel}}} \approx 10^8 \text{ yr } \left( \frac{M_s}{10^6 M_\odot / h} \right)^{1/3} \Delta_s^{-1/3} \left( \frac{v_{\text{rel}}}{200 \text{ km/s}} \right)^{-1}
\]

• Order of magnitude estimates for individual lenses:

\[\nabla \rightarrow \frac{1}{r_s}; \quad \int d\chi \rightarrow r_s; \quad \text{etc.}\]
“Standard” CDM substructure

- Assume substructure made up of halos
- In reality, many subhalos will overlap
- To calculate lensing, we need halo mass and density
- Assume that $\rho(<r_s) \simeq \bar{\rho}(z_f)$, and obtain $z_f$ from spherical collapse
“Standard” CDM substructure

• $z_f$ from spherical collapse: $D(z_f) \sigma(M) = \delta_c$

\[
M = M_{\text{vir}}(M_s)
\]

\[
\Rightarrow \Delta_s \simeq (1 + z_f)^3 \lesssim 10^4
\]

Substructure is very, very diffuse - subhalos are big
Observables
Time delay & Doppler shift

• Apply to “clocks”, e.g. ms pulsars

• **Time delay**: measure *change* in delay due to relative lens-source motion:

\[
\Delta t = -2v_{\text{rel}} \cdot \nabla \int d\chi \Phi = -\frac{\Delta \nu}{\nu}
\]

• **Doppler shift**: acceleration of source & observer due to gravitational pull of lens

\[
\frac{\Delta \nu}{\nu} = \int dt \, \hat{n} \cdot (\nabla \Phi_{\text{src}} - \nabla \Phi_{\text{obs}})
\]

\( \nu \): observed pulsar frequency
Observational prospects

- Time delay:

\[
\frac{\Delta \nu}{\nu} \sim v_{\text{rel}} \Phi \sim 0.1 \frac{\mu\text{s}}{\text{yr}} \left( \frac{M_s}{10^6 M_\odot/h} \right)^{2/3} \Delta_s^{1/3} \left( \frac{v_{\text{rel}}}{300 \text{ km/s}} \right)
\]
Observational prospects

- **Time delay:**

\[
\frac{\Delta \nu}{\nu} \sim \nu_{\text{rel}} \Phi \sim 0.1 \frac{\mu \text{s}}{\text{yr}} \left(\frac{M_s}{10^6 M_\odot/h}\right)^{2/3} \Delta_s^{1/3} \left(\frac{\nu_{\text{rel}}}{300 \text{ km/s}}\right)
\]

- **Doppler shift:**

\[
\frac{\Delta \nu}{\nu} \sim T_{\text{obs}} \frac{c}{\chi_{\text{lens}}} \Phi
\]

\[
\sim 0.03 \frac{\mu \text{s}}{\text{yr}} \left(\frac{T_{\text{obs}}}{\text{yr}}\right) \left(\frac{M_s}{10^6 M_\odot/h}\right)^{2/3} \Delta_s^{1/3} \left(\frac{\chi_{\text{lens}}}{\text{kpc}}\right)^{-1}
\]

- **Doppler shift wins for close lenses and long observing times**
Observational prospects

- Time delay for ms pulsars - Siegel, Hertzberg, Fry (2007)

\[ M_s \sim 10^{-2} M_\odot \]

Unfortunately, density assumed is \( \sim 10^{12} \) times higher than standard estimate...
Observational prospects

- Baghram, Afshordi, Zurek (2011)

Using NL P(k) from stable clustering

Doppler wins for long observing times (>~ 10 years)
Astrometric perturbations

• Measure proper motion of source -> time evolution of deflection angle

\[ \dot{\theta}^i - \dot{\theta}^i_{\text{prop}} = 2\dot{\theta}^j_{\text{lens}} \frac{\partial}{\partial \theta^j} \int d\chi \frac{\chi_s - \chi}{\chi_s} \partial^i_{\perp} \Phi \]

\[ = 2v^j_{\text{lens, } \perp} \int d\chi \frac{\chi_s - \chi}{\chi_s} \partial^j_{\perp} \partial^i_{\perp} \Phi \]

• In fact, need time dependence of this to disentangle from unknown proper motion
Observational prospects

\[ \dot{\theta} \sim \frac{v_{\text{rel}}}{r_s} \Phi \sim 10^{-8} \mu\text{as yr} \left( \frac{M_s}{10^6 M_\odot / h} \right)^{1/3} \Delta_s^{2/3} \]

- More sensitive to density due to additional spatial derivative
- Apply to stars targeted by GAIA
Observational prospects

- Erickcek & Law (2011):

Here, density $\sim 10^5$ times standard estimate
- still not observable...
Observational prospects

- Increasing density further,

\[ \rho \propto \frac{M}{r^\alpha} \]

Increasing density further,

- Can be used to constrain ultra-compact minihalos (UCMH) - Li, Erickcek, Law (2012)
Flux Magnification

\[ \kappa \sim \frac{\chi_{\text{lens}}}{r_s} \Phi \sim 10^{-13} \left( \frac{M_s}{10^6 M_\odot / h} \right)^{1/3} \Delta_s^{2/3} \left( \frac{\chi_{\text{lens}}}{\text{kpc}} \right) \]

- We don’t know intrinsic flux (typically), thus only observe change in magnification:

\[ \dot{F}_{\text{obs}} \sim v_{\text{rel}} \nabla \kappa \sim 10^{-21} \text{yr}^{-1} \Delta_s \left( \frac{\chi_{\text{lens}}}{\text{kpc}} \right) \]

- Very small, but mass-independent and proportional to density

- This is why we can look for planets with microlensing...
Observational prospects

- Griest et al (2011): forecasted constraints on DM made of low-mass black holes, using Kepler photometric data

- Very low masses constrained - possible because BH are very dense
Conclusions
Conclusions

• It is natural to ask whether DM substructure within the Milky Way can be detected through lensing.

• The main obstacle for standard CDM is that substructure is diffuse: $\rho < 10^4 \bar{\rho}$.

• Observables roughly scale as
  • Time delay & Doppler: $\propto \rho^{1/3}$
  • Astrometry: $\propto \rho^{2/3}$
  • Magnification: $\propto \rho$
Conclusions

• Thus, for standard CDM, time delay and Doppler are most promising - but probably still out of reach even for futuristic experiments

• If DM substructure is much denser, then astrometry and magnification can become interesting
  • Astrometry constrains ultra-compact minihalos
  • Kepler constrains BH DM

• More observables still to be explored in this direction!